Topic: Chain rule with product rule

Question: Use product rule and chain rule to find the derivative.

$$y = (4x - 7)^2 (2x + 3)$$

Answer choices:

A $y' = (4x - 7)\left(12x + 5\right)$

B $y' = 2\,(4x - 7)\,(2x + 3)$

C $y' = 2\,(4x - 7)\left(12x + 5\right)$

D $y' = 2\,(4x - 7)^3\left(12x + 5\right)$

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Solution: C

To take the derivative of this function, we'll need to apply product rule.

$$y' = (4x-7)^2 \frac{d}{dx}(2x+3) + (2x+3)\frac{d}{dx}(4x-7)^2$$

Now, to take the derivative of $(4x-7)^2$, we'll need to apply chain rule. Remember to take the derivative of the outside function first, leaving the inside function completely untouched, then multiply by the derivative of the inside function. Here, our outside function is u^2, where $u = 4x-7$, and our inside function is $4x-7$.

$$y' = (4x-7)^2(2) + (2x+3)2(4x-7)\frac{d}{dx}(4x-7)$$

$$y' = (4x-7)^2(2) + (2x+3)2(4x-7)(4)$$

$$y' = 2(4x-7)^2 + 8(2x+3)(4x-7)$$

$$y' = 2(4x-7)\left[(4x-7) + 4(2x+3)\right]$$

$$y' = 2(4x-7)\left[4x-7+8x+12\right]$$

$$y' = 2(4x-7)\left(12x+5\right)$$

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Topic: Chain rule with product rule

Question: Use product rule and chain rule to find the derivative.

$$f(x) = 2x^2e^{3x}$$

Answer choices:

A $f'(x) = 12xe^{3x}$

B $f'(x) = 24x^3e^{6x}$

C $f'(x) = 4xe^{3x} + 6x^2e^{3x}$

D $f'(x) = 12xe^{3x} + 2x^2e^{3x}$

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Solution: C

First, note that $f(x) = 2x^2e^{3x}$ is the product of two functions of x. Therefore, we'll need to use the product rule to differentiate $f(x)$:

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

In this case, let $g(x) = 2x^2$ and $h(x) = e^{3x}$. Next, we need to find the derivatives $g'(x)$ and $h'(x)$. Finding $g'(x)$ is straightforward:

$$g(x) = 2x^2$$

$$g'(x) = 2(2x) = 4x$$

However, note that $h(x)$ is actually a composite function. If we let $j(u) = e^u$ and $u = k(x) = 3x$, we'll find that $h(x) = j(k(x))$.

Therefore, we'll need to use the chain rule to differentiate $h(x)$:

$$h(x) = j(k(x))$$

$$h'(x) = j'(k(x)) \cdot k'(x)$$

Let's find the derivatives of j and k to substitute in:

$$j(u) = e^u$$

$$j'(u) = e^u$$

$$j'(k(x)) = e^{3x}$$

$$k(x) = 3x$$

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$$k'(x) = 3$$

Therefore:

$$h'(x) = j'(k(x)) \cdot k'(x) = e^{3x} \cdot 3 = 3e^{3x}$$

Now we can use the information we've found to substitute into our original formula to find $f'(x)$:

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$f'(x) = 4x \cdot e^{3x} + 2x^2 \cdot 3e^{3x}$$

$$f'(x) = 4xe^{3x} + 6x^2e^{3x}$$

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Topic: Chain rule with product rule

Question: Use product rule and chain rule to find the derivative.

$$y = \sin 3x \cos x^2$$

Answer choices:

A $\dfrac{dy}{dx} = -6x \sin 3x \cos x^2$

B $\dfrac{dy}{dx} = \sin 3x \cos x^2 - 6x \cos 3x \sin x^2$

C $\dfrac{dy}{dx} = -6x \cos 3x \sin x^2$

D $\dfrac{dy}{dx} = 3 \cos 3x \cos x^2 - 2x \sin 3x \sin x^2$

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Solution: D

First, note that $y = \sin 3x \cos x^2$ is the product of two functions of x. If we let $v = \sin 3x$ and $w = \cos x^2$, then $y = v \cdot w$. Therefore, we will need to use the product rule to differentiate the function:

$$y = v \cdot w$$

$$\frac{dy}{dx} = \frac{dv}{dx} \cdot w + v \cdot \frac{dw}{dx}$$

To do so, we'll need to find dv/dx and dw/dx. Note that v is a composite function ("function of a function"): $v = f(u) = f(g(x))$ where $f(u) = \sin u$ (the "outer function") and $u = g(x) = 3x$ (the "inner function"). Therefore, finding the derivative dv/dx involves using the chain rule:

$$\frac{dv}{dx} = \frac{dv}{du} \cdot \frac{du}{dx}$$

$$\frac{dv}{dx} = \cos u \cdot 3$$

$$\frac{dv}{dx} = 3 \cos 3x$$

Similarly, w is also a composite function ($w = \cos z$ where $z = x^2$), and so finding dw/dx also involves the chain rule:

$$\frac{dw}{dx} = \frac{dw}{dz} \cdot \frac{dz}{dx}$$

$$\frac{dw}{dx} = -\sin z \cdot 2x$$

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$$\frac{dw}{dx} = -2x \sin x^2$$

Now we have the pieces we need to find dy/dx:

$$\frac{dy}{dx} = \frac{dv}{dx} \cdot w + v \cdot \frac{dw}{dx}$$

$$\frac{dy}{dx} = 3 \cos 3x \cdot \cos x^2 + \sin 3x \cdot -2x \sin x^2$$

$$\frac{dy}{dx} = 3 \cos 3x \cos x^2 - 2x \sin 3x \sin x^2$$

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